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HEAT TRANSFER IN GEOMETRICALLY SIMILAR CYLINDERS

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SUMMARY

The power- and heat-stress conditions of geometrically similar engines are discussed. The advantages accruing from smaller cylinder dimensions are higher specific horsepower, lower weight per horsepower, lower piston temperature, and less frontal area, with reduced detonation tendency.

INTRODUCTION

Aside from the attempts toward units of high power, the principal aims in aircraft-engine development are directed toward lower specific horsepower and lower head resistance. In an engine of given design form, these quantities are affected by the size of the individual cylinder, that is, by the type of subdivision of the total swept volume of the engine. In the following the characteristics of geometrically similar cylinders and engines built up of such cylinders are explored from the standpoint of horsepower and heat transfer.

POWER

Notation

P_i force

m_i mass

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*"Leistung und Wärmeabfuhr bei geometrisch ähnlichen Zylindern." Jahrbuch 1938 der deutschen Luftfahrtforschung, pp. II 80-II 82.

n_i rpm

r_i crank radius

N_i power

p_{mi} mean effective pressure

F_i piston area

V_{hi} swept volume

s_i stroke

The general power equation reads:

$$N = \frac{p_m F s n}{C} = \frac{p_m V_h n}{C} \tag{1}$$

The power increases with increasing mean effect pressure, greater swept volume, and rising rpm, and is, in addition, dependent upon quantity C, which compromises the engine cycle - two or four stroke. The mean pressure p_m is largely determined by the fuel quantity combustible in the inducted air volume conformably to chemical laws; it can be increased by supercharging the engine. A graphical view of these conditions is shown in figure 1: the straight lines with stroke s as parameter, the abscissa c_m and the ordinate form one family, the hyperbolas with N/V_h as parameter; abscissa p_m and n form another. Proceeding from the mean piston speed $c_m = \frac{2rn}{30} = \frac{sn}{30}$ as indication for the flow velocity of the gases through the valves and hence for the charge of the cylinder, the stroke s defines the rotational speed n. To illustrate: For $c_m = 16$ meters per second with $s = 60$ millimeters, the speed is $n = 8000$ rpm. The rotational speed in conjunction with the obtainable mean pressure, then, defines the specific horsepower $\frac{N}{V_h} = \frac{p_m n}{C}$. At $n = 8000$ rpm and $p_m = 10$ kg/cm^2, the specific horsepower is 90 horsepower per liter for the four-stroke cycle and 180 horsepower per liter for the two-stroke.

An arbitrary rise of V_h and n at once is not obtainable for mechanical reasons and because of the inferior cylinder charge resulting from the flow resistance in the gas passages. In order that these conditions may be surveyed two geometrically similar cylinders (reference 1) with linear dimensions l_o and l_1 must be considered. When equal admissible specific pressures are postulated on piston, bearings, etc., the proportional value for the effective forces P_i reads:

$$\frac{P_1}{P_o} = \frac{l_1{}^2}{l_o{}^2} = \lambda^2 \quad \text{with} \quad \lambda = \frac{l_1}{l_o} \tag{2}$$

By equal course of combustion process in both cylinders, the bearing load due to the ignition pressures is the same at all revolutions per minute; whereas, the bearing loads due to mass forces increase as the square with the revolutions per minute and therefore become at aspiration, that is, no-combustion pressures, decisive for the permissible bearing pressures. Thus (2) affords

$$\frac{P_1}{P_o} = \frac{m_1 r_1 \ n_1{}^2}{m_o r_o n_o{}^2} = \lambda^2 \tag{3}$$

Hence, because of the stipulated geometrical similitude,

$$\frac{n_1{}^2}{n_o{}^2} = \lambda^2 \ \frac{m_o}{m_1} \ \frac{r_o}{r_1} = \lambda^2 \ \frac{1}{\lambda^3} \ \frac{1}{\lambda} = \frac{1}{\lambda^2}$$

and

$$\frac{n_1}{n_o} = \frac{1}{\lambda} \tag{4}$$

The limit revolutions per minute are in the inverse ratio of the linear dimensions. The mean piston speeds decisive for the cylinder charge have the proportional value

$$\frac{c_{m_1}}{c_{m_o}} = \frac{s_1 n_1}{s_o n_o} = \lambda \ \frac{1}{\lambda} = 1 \tag{5}$$

that is, they are equal and condition equal charge. From the general power equation (1) there follows for equal p_m

$$\frac{N_1}{N_o} = \frac{V_{h_1} n_1}{V_{h_o} n_o} = \lambda^3 \ \frac{1}{\lambda} = \lambda^2 \tag{6}$$

The power increases as the square of the length ratio. Thus the power referred to unit of swept volume is as follows:

$$\frac{N_1}{V_{h_1}} \frac{V_{h_0}}{N_0} = \lambda^2 \frac{1}{\lambda^3} = \frac{1}{\lambda} \qquad (7)$$

The specific horsepower units are in the inverse ratio of the linear dimensions; hence they increase with decreasing linear dimensions.

For the weight per unit of horsepower (kg/hp) the equation reads:

$$\frac{G_1}{N_1} \frac{N_0}{G_0} = \lambda^3 \frac{1}{\lambda^2} = \lambda \qquad (8)$$

With geometrically similar design the weight per horsepower (kg/hp) increases with the linear dimensions.

Concerning the previous assumption that the constancy of piston speed is decisive solely for the identical charge of similar cylinders, it is to be noted that, strictly speaking, the flow in the gas passages is similar only with constant Reynolds number $R_e = \frac{wl}{\nu}$. However, the resultant effect of l is relatively small. In practice, moreover, the reduction of weight per horsepower unit is no longer possible in the ratio λ when changing to very small cylinder dimensions, since the wall thickness of the cylinders or perhaps the spacing and thickness of the cooling fins cannot be arbitrarily reduced. The extent of the reduction in weight per horsepower unit is therefore less than that which corresponds to the length ratio λ.

HEAT REMOVAL (Fig. 2)

Here

$F(\varphi, l)$ denotes the amount of inside area of the cylinder at crank angle φ

$T_i(\varphi)$ gas temperature at crank angle φ

T_a outside temperature of coolant

n engine revolutions per minute

$k(\varphi, l, n)$ coefficient of heat transfer at crank angle φ

A cylinder of the geometrically similar bank transfers, at crank setting φ in time interval dt, to the coolant the heat volume

$$dQ = F(\varphi, l)k(\varphi, l, n)[\tau_i(\varphi) - \tau_a]dt \tag{9}$$

With $d\varphi = \frac{\pi n}{30}\,dt$, the heat dissipation per cycle is, on the assumption of independence of temperature τ_i and τ_a from l and n:

$$\bar{Q} = \frac{30}{\pi n}\int_{\varphi=0}^{\varphi=a2\pi} F(\varphi, l)k(\varphi, l, n)[\tau_i(\varphi) - \tau_a]d\varphi \tag{10}$$

For the two-stroke cycle, a = 1, for the four-stroke, a = 2. Referred to time unit, the volume of heat removed amounts to $\frac{n}{a}$ times; hence

$$Q = \frac{30}{a\pi}\int_{\varphi=0}^{\varphi=a2\pi} F(\varphi, l)k(\varphi, l, n)[\tau_i(\varphi) - \tau_a]d\varphi \tag{11}$$

Theoretically it is therefore necessary to differentiate between the heat removal per cycle and per unit time.

If the same cylinder is considered at different revolutions per minute and the coefficient of thermal conductivity $k(\varphi, l, n)$ is assumed to be unaffected by n, the heat volume $\bar{Q}$ transferred to the coolant per cycle decreases by increasing n and the heat transfer Q per unit time remains the same. But, since the power N, by equal mean pressure p_m, increases linearly with increasing n together with the quantity of fuel inducted per unit time, this means that a continuously decreasing portion of the residuary heat not converted into power passes into the coolant and an increasing portion into the exhaust gases. In practice, however, $k(\varphi, l, n)$ increases with increasing n, because of the improved coefficient of heat transfer on the gas side, as is explained later on. Inasmuch as the rise is slower than that of n, the heat removal Q per unit time is no longer invariable, although a displacement of the residuary heat in favor of the exhaust heat is still always present.

Proceeding with the conditions on different geometrically similar cylinders, the functions F and k can be split in two parts, one of which comprises the effect of crank setting, the other the geometrical dimensions and n; hence

$$\left.\begin{aligned} F(\varphi, l) &= l^2 f(\varphi) \\ k(\varphi, l, n) &= k(l, n)\ k(\varphi) \end{aligned}\right\} \tag{12}$$

Then equation (11) gives

$$Q = l^2 k(l,n) \frac{30}{a\pi} \int_{\varphi=0}^{\varphi=a2\pi} f(\varphi) k(\varphi) [\tau_i(\varphi) - \tau_a] d\varphi \tag{13}$$

For two similar cylinders the value of the integral is equal, giving the proportional value

$$\frac{Q_1}{Q_0} = \lambda^2 \frac{k(l_1, n_1)}{k(l_0, n_0)}$$

or, for the amount of heat removal per unit surface,

$$\frac{q_1}{q_0} = \frac{k(l_1, n_1)}{k(l_0, n_0)} \tag{14}$$

The coefficient of thermal conductivity $k(l,n)$ is

$$k(l,n) = \frac{1}{\dfrac{1}{\alpha_G(l,n)} + \dfrac{1}{\alpha_K} + \dfrac{\delta(l)}{\lambda_W}} \tag{15}$$

where $\alpha_G(l,n)$ denotes the average value of the heat transfer coefficient on the gas side per cycle, α_K the heat transfer coefficient on the coolant side, referred to the smooth cylinder surface, λ_W the coefficient of heat conduction of the wall, and $\delta(l)$ the wall thickness. (α_K and λ_W are largely unaffected by l and n). For the determination of $\alpha_G(l,n)$ the similitude relation

$$N_u = \Phi(P_e)$$

with Nusselt's constant $N_u = \dfrac{\alpha_G l}{\lambda_G}$ and Péclet's constant

$$P_e = \frac{w_G l}{\alpha_G} = \frac{w_G l c_G \gamma_G}{\lambda_G}$$

are resorted to, where $\alpha_G = \frac{\lambda_G}{c_G \gamma_G}$ is the temperature conductivity factor with λ_G as heat conductivity factor of the gas, c_G as specific heat of gas, γ_G as specific weight of gas, and w_G as a characteristic speed of the gas content. All quantities are averages over a cycle. Function Φ can be expressed as power function

$$N_u = C'(P_e)^{\mu} \tag{16}$$

or

$$\alpha_G = C' \frac{\lambda_G}{l}\left(\frac{w_G l c_G \gamma_G}{\lambda_G}\right)^{\mu} = C' w_G^{\mu} \left(\frac{l}{\lambda_G}\right)^{\mu-1} (c_G \gamma_G)^{\mu}$$

Nusselt's* experimental value of $\mu = 0.8$ for the heat transfer in a straight pipe of circular section serves as a basis of the present study. Then α_G increases by the assumedly constant values c_G, γ_G, and λ_G by equal cylinder with increasing w_G, hence increasing n. But the increase by $\mu = 0.8$ is slower than that of n. Assuming equal λ_G, c_G, and γ_G for different cylinders also affords

$$\frac{\alpha_{G_1}}{\alpha_{G_0}} = \frac{l_0}{l_1}\left(\frac{w_{G_1} l_1}{w_{G_0} l_0}\right)^{\mu} = \left(\frac{w_{G_1}}{w_{G_0}}\right)^{\mu} \lambda^{\mu-1} \tag{17}$$

The piston speed can be dealt with as characteristic speed and gives with $\frac{w_{G_1}}{w_{G_0}} = 1$

$$\frac{\alpha_{G_1}}{\alpha_{G_0}} = \lambda^{\mu-1} = \lambda^{-0.2} \tag{18}$$

From this it is apparent that for $\mu < 1$, that is, practically $\mu = 0.8$, the heat transfer coefficient on the gas side becomes smaller with increasing cylinder dimensions. Also, $\delta(l)$ becomes greater by greater cylinder dimensions. According to (14) and (15), the heat transfer through the wall to the coolant per unit surface and unit time therefore decreases on larger cylinders.

*See reference 2.

For equal piston speed, the larger cylinder gives off a little less heat per unit surface than the smaller cylinder because of the longer heat path. Since the area, as well as the performance, and hence the inducted fuel volume in similar cylinders, increases in the ratio λ^2 the heat volume inducted in the cylinder per unit cooling surface is always identically great. The proportion of the cooling heat on the total heat accordingly drops a little with increasing cylinder size, while the exhaust heat rises. The same effect was observed above for the identical cylinder by increasing n. It generally occurs on geometrically similar cylinders, when the piston speed rises (where, however, the principle of similitude is broken). As the charge by high piston speeds can be maintained only by supercharging, this possibility of raising the piston speed is greater with the small cylinder because of the lower detonation tendency. In this respect the small cylinder can also become superior in respect to its cooling input per horsepower-hour as proved by experiments on geometrically similar cylinders.

Although, according to the foregoing, the heat removal of the large cylinder is lower by equal piston speed, the temperature conditions act in its disfavor, as may be seen from the ensuing study of heat removal on the piston. For simplicity, assume that each unit area of the piston bottom receives the same amount of heat for unit time and that the total heat flow in the piston bottom is in radial direction, the axial flow being discounted. Then the conditions in this substitute system are exactly the same as in an infinitely long homogeneous cylinder of radius R, in which the heat volume W per unit space and time is produced and at whose jacket with heat transfer coefficient α_K the coolant of temperature τ_K borders. The temperature distribution in this cylinder satisfies the linear differential equation of the second order

$$\frac{d^2 \tau}{d r^2} + \frac{1}{r} \frac{d \tau}{d r} + \frac{W}{\lambda_W} = 0 \tag{19}$$

the solution of which, with the given boundary conditions, is (reference 2)

$$\tau = \tau_K + \frac{W R^2}{4 \lambda_w} \left[1 + \frac{2\lambda_W}{\alpha_K R} - \left(\frac{r}{R} \right)^2 \right] \tag{20}$$

The temperature in the cylinder axis is accordingly

$$\tau_A = \tau_K + \frac{W\ R^2}{4\ \lambda_W}\left(1 + \frac{2\lambda_W}{\alpha_K R}\right) \tag{21}$$

Now $\frac{2\lambda_W}{\alpha_K R} \gg 1$; hence within the scope of the study, it approximates to

$$\tau_A = \tau_K + \frac{W\ R}{2\alpha_K} \tag{22}$$

The heat volume in the piston per unit surface with which W is identical, decreases slowly with increasing cylinder size according to previous arguments. Since R rises in proportion to increasing cylinder size, $\frac{W\ R}{2\alpha_K}$ and hence τ_A increase also with the size of the cylinder (by equal coolant temperature τ_K). The piston in large cylinders becomes accordingly hotter than in small cylinders, as proved by tests.(piston cooling on large cylinders).

POWER AND COOLING CONDITIONS OF A COMPLETE ENGINE

An engine with specified power N can be designed either for high speed with small stroke volume or slow speed with large stroke volume. If the revolutions per minute are pushed to the limits imposed by the mechanical stresses and the flow resistances, the performances of the individual cylinders are as λ^2, according to equation (6). Hence it requires a proportional number of cylinders λ^{-2} by equal total N. The cooling surfaces of the separate cylinders are in the ratio λ^2; hence for λ^{-2} cylinders the same cooling surface is available. The heat given off on the coolant by the engine with large cylinder is therefore somewhat less than from the high-speed engine with small cylinders because of the smaller heat removal by larger cylinders. But of substantially negative influence on the engine operation are the higher piston temperatures and the greater detonation tendency. Since the total stroke volume and the total weight of the cylinders on different engines have the ratio λ, the same holds true for the weight per horsepower and the horsepower per cubic inch, as on the separate cylinder; that is, both quantities are more favorable with smaller dimensions and more cylinders than with large individual cylinders. The frontal areas, decisive for the air resistance, are also more favorable in the ratio λ^2 on the smaller engines.

Translation by J. Vanier,
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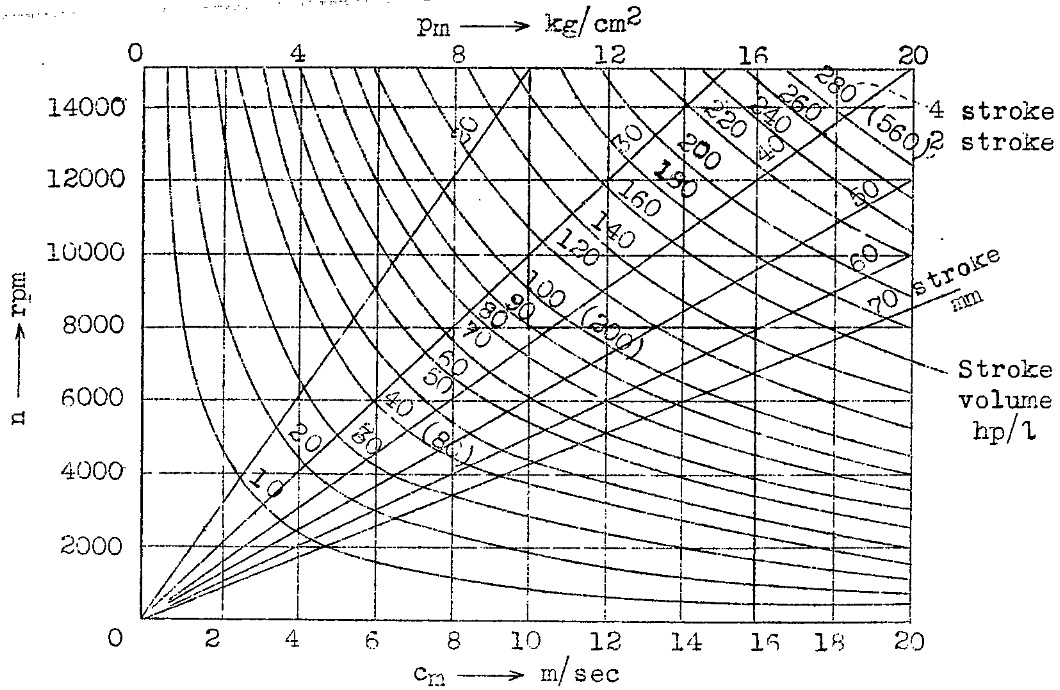

Figure 1.- Characteristic curves of reciprocal engines.

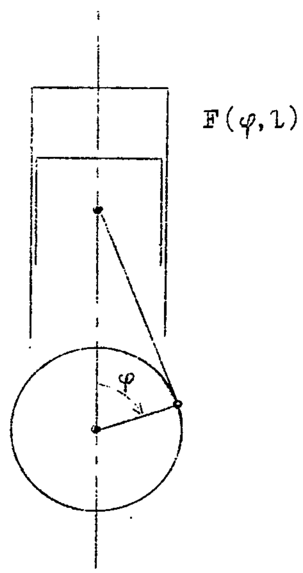

Figure 2.-